**3. INTEGRATION**

**First fundamental theorem of integral calculus:**

If f(x) is continuous [a,b] then the function F(x) = is,

1) Continuous [a,b]

2) differentiable (a,b)

3)

**Second fundamental theorem of integral calculus (Area Under Curve):**

If f(x) is continuous [a,b] and F(x) is antiderivative of f(x) then,

**Mean Value theorem of integral:**

If f(x) is continuous [a,b] then there exist a real number C ϵ (a,b) such that

**Properties of definite integrals:**

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**Improper integrals:**

**First Kind (Limits infinite):**  is said to be Improper integral if a = **∞** **or** b **= -∞** **or** both.

**Second Kind (Function infinite):**  is said to be Improper integral if a and b are finite but f(x) is infinite for some x ϵ (a, b).

**Comparison Test:**

1) If 0 ≤ f(x) ≤ g(x) for all x ϵ [a, b] and converges than also converges.

2) If 0 ≤ f(x) ≤ g(x) for all x ϵ [a, b] and diverges than also diverges.

**Improper Integrals for First Kind Test:** If f(x) and g(x) are two positive functions such that then and both converges or diverges together.

From the definition of limits,

There exists some N (x > N) such that ==> f(x) < (K+1) g(x)

**, Where is finite and = Finite.**

**Improper Integrals for Second Kind Test:** If f(x) and g(x) are two positive functions and,

1) f(x) → ∞ as x → a such that

2) f(x) → ∞ as x → b such that

Then, and both converges or diverges together.

**Gamma Function:**

,

**Beta Function:**

And And

**ILATE: Inverse => Log => Arithmetic => Trigonometric => Exponential**

The length of the arc y = f(x) between x = a and x = b is given by

The volume of the solid generated by revolving the area bounded by the curve y = f(x), x-axis and the x = a and x = b about y-axis is

The region R enclosed by curves y1 = f(x) and y2 = g(x) is rotated about the x-axis. the volume of the resulting solid

**Leibnitz Formula:**

**Integral as sum of the limit:**

If f(x) is continuous in the interval [a, b] then the definite integral of f(x) with limits a and b is defined by the equation

To express a given series as definite integral:

1) Write the general term i.e. i.e.

2) Replace by x and by dx

3) integrate the resulting expression taking the **lower limit** = where, r is as in the first term And **Upper limit** = where, r is as in the last term

**Double Integrals:**

**Application of Double Integrals:**

**Volume using double integrals:**

The volume V beneath the surface z = f(x, y) > 0 and above the region R in the xy-plane is

**Triple Integrals:**

Note: is continuous over the closed region bounded by surfaces in 3D space

**Application of Triple Integrals:**

**Change of Variable:**

Where J = Jacobian =

For Polar coordinate, J = R And x = R cos θ, y = R sin θ

For Cylindrical coordinate, J = R And x = R cos θ, y = R sin θ, z = z

For Spherical coordinate, J = R2 sin θ, And x = R sin θ cos φ, y = R sin θ sin φ, z = R cos θ